Homework Set #6

Problem 1

Given the normally distributed independent random variables

$$X_i \sim N(\mu_i, \sigma_i^2)$$

compute the first- and second-order Sobol indices for the functions below.

(a) $Y = X_1 + X_2$

(b) $Y = X_1 X_2$

(c) $Y = X_1 + X_2 + X_1 X_2$

Problem 2

Consider the limit state function in Homework Set 4 Problem 1, i.e.

$$G(\boldsymbol{X}) = X_1^4 + 2X_2^4 - 20$$

where $X_1 \sim N(10, 5^2)$ and $X_2 \sim N(10, 5^2)$ are independent. Provide the main- and total-effect indices of the below functions for X_1 and X_2, and discuss why the results from (a),(b), and (c) are similar/different.

(a) Nonlinear limit state function $Y = G(\boldsymbol{X})$

(b) Linearized limit state function $Y = G_{FORM}(\boldsymbol{Z})$, where $\boldsymbol{Z}$ is standard normal random variables, i.e, $Z_i = F_Z^{-1}(F_{X_i}(X_i))$

(c) Failure index function $Y = \mathbb{1}(G(\boldsymbol{X}) \leq 0) \simeq \mathbb{1}(G_{FORM}(\boldsymbol{Z}) \leq 0)$

Problem 3

Perform global sensitivity analysis using your own model:

- Analytical model, finite element model, regression model, physical model, etc. are all accepted. However, the model should have more than one input variable.

- Input-output dataset can also be used instead of a model.

- In case it is difficult to find any model, try the OpenSees/Python models in the quoFEM user manual (link) or this Matlab borehole function (https://uqworld.org/t/borehole-function/60)

Provide reasonable assumptions for the probability distribution of input variables and run global sensitivity analysis. A matlab script to run probability model-based GSA is provided, but any other global sensitivity analysis algorithm/tool (quoFEM, self-written program, analytic derivation, toolboxes such as UQpy or UQlab, etc.) can be used.

(a) Briefly describe your model and assumptions on input random variables.

(b) What algorithm/tool did you choose for the global sensitivity analysis?

(c) What was the expected outcome of the global sensitivity analysis?

(d) Compute main Sobol index values. Are the results similar to your intuition in (c)? If not, discuss why.